Lecture 11

Terrestrial infrared radiative processes. Part 4:

Infrared radiative transfer revisited. IR radiative heating/cooling rates

Objectives:

1. IR radiative transfer revisited.
2. Infrared radiative heating/cooling rates.
3. Concept of broadband flux emissivity.

Required reading:

L02: 4.2.2; 4.5-4.7

1. IR radiative transfer revisited.

Recall Lecture 8 where we have derived the solutions of the radiative transfer equation for the **monochromatic upward and downward intensities** in the IR for a plane-parallel atmosphere consisting of absorbing gases (no scattering)

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)\exp\left(-\frac{\tau^*-\tau}{\mu}\right) + \frac{1}{\mu}\int_\tau^{\tau^*}\exp\left(-\frac{\tau'-\tau}{\mu}\right) B_\nu(\tau')\,d\tau' \qquad [8.3a]$$

$$I_\nu^{\downarrow}(\tau;-\mu) = \frac{1}{\mu}\int_0^{\tau}\exp\left(-\frac{\tau-\tau'}{\mu}\right) B_\nu(\tau')\,d\tau' \qquad [8.3b]$$

and in terms of transmission function

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)T_\nu(\tau^*-\tau;\mu) - \int_\tau^{\tau^*} B_\nu(\tau')\frac{dT_\nu(\tau'-\tau;\mu)}{d\tau'}\,d\tau' \qquad [8.4a]$$

$$I_\nu^{\downarrow}(\tau;-\mu) = \int_0^{\tau} B_\nu(\tau')\frac{dT_\nu(\tau-\tau';\mu)}{d\tau'}\,d\tau' \qquad [8.4b]$$

Recall that Eq.[8.3a, b] and Eq.[8.4a, b] have been derived the whole atmosphere with the optical depth τ_v* for two boundary conditions:

Surface: assumed to be a blackbody in the IR emitting with the surface temperature T_s,

$$I_v^{\uparrow}(\tau_v^*, \mu) = B_v(T_s) = B_v(T_s(\tau_v^*)) = B_v(\tau_v^*)$$

Top of the atmosphere (TOA), $\tau_v = 0$: no downward emission

$$I_v^{\downarrow}(0, -\mu) = 0$$

In Lecture 2, the upwelling and downwelling fluxes were defined as

$$\boxed{\begin{aligned} F_v^{\uparrow} &= 2\pi \int_0^1 I_v^{\uparrow}(\mu)\mu\, d\mu \\ F_v^{\downarrow} &= 2\pi \int_0^1 I_v^{\downarrow}(-\mu)\mu\, d\mu \end{aligned}} \qquad [11.1]$$

NOTE: Eq.[11.1] assumes that there is no dependency on ϕ in a plane-parallel atmosphere.

Thus, we can re-write the radiative transfer equation and its solutions in terms of **the monochromatic upward and downward fluxes.** From Eq.[8.3a, b], we have

$$\boxed{\begin{aligned} F_v^{\uparrow}(\tau) &= 2\pi B_v(\tau^*)\int_0^1 \exp(-\frac{\tau^* - \tau}{\mu})\mu\, d\mu \\ &+ 2\pi \int_0^1 \int_{\tau}^{\tau^*} \exp(-\frac{\tau' - \tau}{\mu})\; B_v(\tau')\, d\tau' d\mu \end{aligned}} \qquad [11.2a]$$

and

$$\boxed{F_v^{\downarrow}(\tau) = 2\pi \int_0^1 \int_0^{\tau} \exp(-\frac{\tau - \tau'}{\mu})\; B_v(\tau')\, d\tau' d\mu} \qquad [11.2b]$$

Let's introduce **the transmission function** for the radiative flux (called **diffuse transmission function** or **slab transmission function or flux transmission function**) as

$$T_{\nu}^{f}(\tau) = 2\int_{0}^{1} T_{\nu}(\tau;\mu)\mu d\mu \qquad [11.3]$$

where $\boldsymbol{T_{\nu}(\tau;\mu)}$ is the monochromatic transmittance defined in Lecture 7, Eq.[7.2]

Spectral diffuse transmission function (or **transmittance) may be defined as:**

$$T_{\bar{\nu}}^{f}(\tau) = 2\int_{0}^{1} T_{\bar{\nu}}(\tau;\mu)\mu d\mu \qquad [11.4]$$

Using the definition of monochromatic diffuse transmittance and solution of the radiative transfer equation expressed via **the transmittance** Eq.[8.4a, b], the solution for fluxes can be written as

$$\boxed{F_{\nu}^{\uparrow}(\tau) = \pi B_{\nu}(\tau^{*}) T_{\nu}^{f}(\tau^{*}-\tau) - \int_{\tau}^{\tau^{*}} \pi B_{\nu}(\tau') \frac{dT_{\nu}^{f}(\tau'-\tau)}{d\tau'} d\tau'} \qquad [11.5a]$$

and

$$\boxed{F_{\nu}^{\downarrow}(\tau) = \int_{0}^{\tau} \pi B_{\nu}(\tau') \frac{dT_{\nu}^{f}(\tau-\tau')}{d\tau'} d\tau'} \qquad [11.5b]$$

NOTE: On the right side of Eq.[11.5a] for the upward flux, the first term gives the surface emission that is attenuated to the level $\boldsymbol{\tau}$ and the second term gives the emission from the atmospheric layers characterized by the Planck function multiplied by the **weighting function** $dT_{\nu}^{f}/d\tau$. Likewise, the downward flux at a given layer (Eq.[11.5b]) is produced by the emission from the atmospheric layers.

2. Infrared radiative heating/cooling rates.

- Radiative processes may affect the dynamics and thermodynamics of an atmosphere through the generation of **radiative heating/cooling rates**.

NOTE: The thermodynamic equation for the temperature changes in the atmosphere (i.e. the first law of thermodynamic for moist air) includes **the radiative energy exchange term (i.e. total radiative heating/cooling rates** which are solar plus infrared heating/cooling rates). In this lecture we discuss IR radiative rates only (solar will be discussed later in the course).

Let's introduce the **monochromatic net flux** (net power per area at a given height defined as

$$\boxed{F_\nu(z) = F_\nu^{\uparrow}(z) - F_\nu^{\downarrow}(z)} \qquad [11.6]$$

Also we can define **total net flux:**

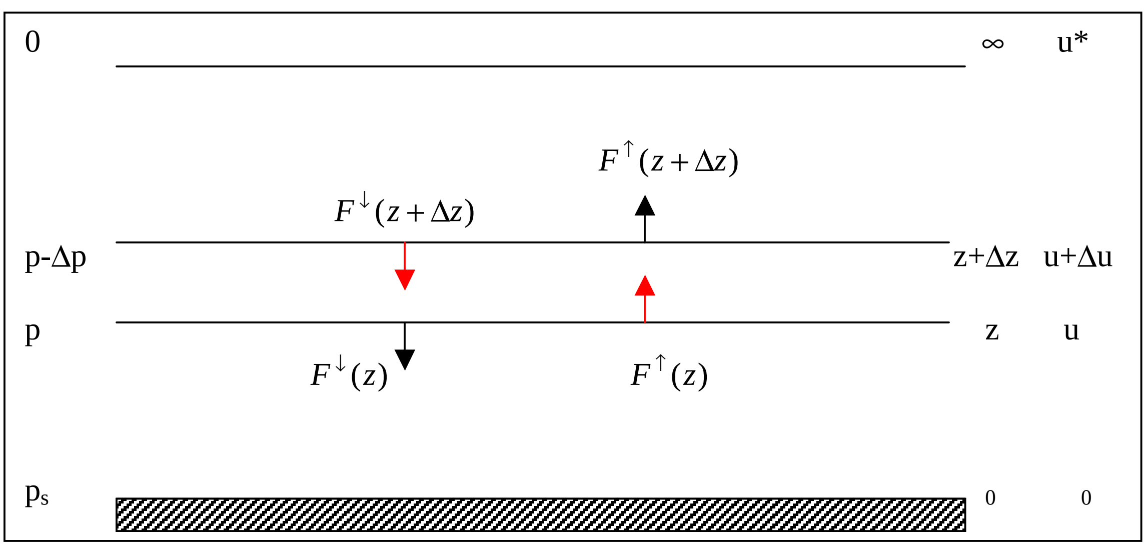
$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z) \qquad [11.7]$$

Introducing the net flux ***F*(z+Δz)** at the level **z+Δz**, we find the **net flux convergence** for the layer **Δz** is

$$\Delta F = F(z + \Delta z) - F(z)$$

***F*(z+Δz) < *F*(z) (hence ΔF < 0) => a layer gains radiative energy => heating**

***F*(z+Δz) > *F*(z) (hence ΔF > 0) => a layer losses radiative energy => cooling**



The IR **radiative heating (or cooling) rate** is defined as the rate of temperature change of the layer ***dz*** due to IR radiative energy gain (or loss):

$$\boxed{\left(\frac{dT}{dt}\right)_{IR} = -\frac{1}{c_p \rho}\frac{dF_{net}}{dz} = \frac{g}{c_p}\frac{dF_{net}}{dp}} \qquad [11.8]$$

where c_p is the specific heat at the constant pressure (c_p = 1004.67 J/kg/K) and ρ is the air density in a given layer.

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EXAMPLE Calculate longwave cooling at night for an atmospheric layer from 0 to 1 km using the upwelling and downwelling fluxes calculated with MODTRAN for US Standard Atmosphere 1976.

Altitude (km)	IR Upwelling flux (W/m^2)	IR Downwelling flux (W/m^2)
0	390	285
1	375	250

SOLUTION:

Need to find net fluxes at each altitude

$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z)$$

At 0 km: F_{net} = 390 – 285 = 105 W/m^2

At 1 km: F_{net} = 375 – 250 = 125 W/m^2

Thus ΔF= 20 W/m^2

$$\left(\frac{dT}{dt}\right)_{IR} = -\frac{1}{c_p \rho}\frac{dF_{net}}{dz} = \frac{-20\ Js^{-1}m^{-2}}{(1.17\ kg/m^3)(1004\ Jkg^{-1}K^{-1})(1000\ m)}$$

dT/dt = -1.7x10^{-5} K/s = -1.5 K/day

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To calculate the IR downward and upward fluxes one needs to know:

i) Atmospheric characteristics: vertical profiles of T, P and air density

ii) The vertical profiles of IR radiatively active gases, clouds and aerosols.

To calculate the IR heating/cooling rates one needs to know:

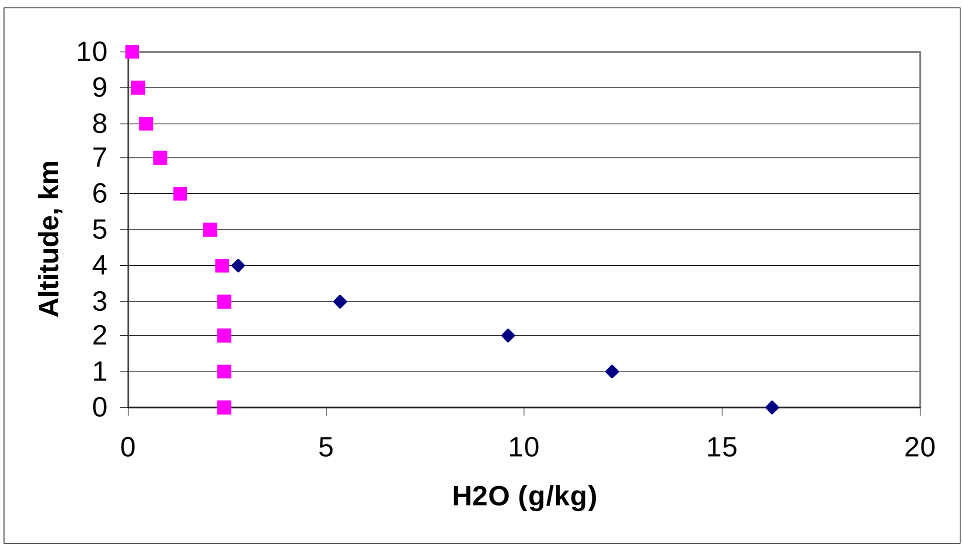
i) Profiles of IR upwelling and downwelling fluxes (to calculate the profile of the IR net fluxes);

ii) Using the profile of net fluxes and air density, one calculates the IR radiative heating/cooling rates

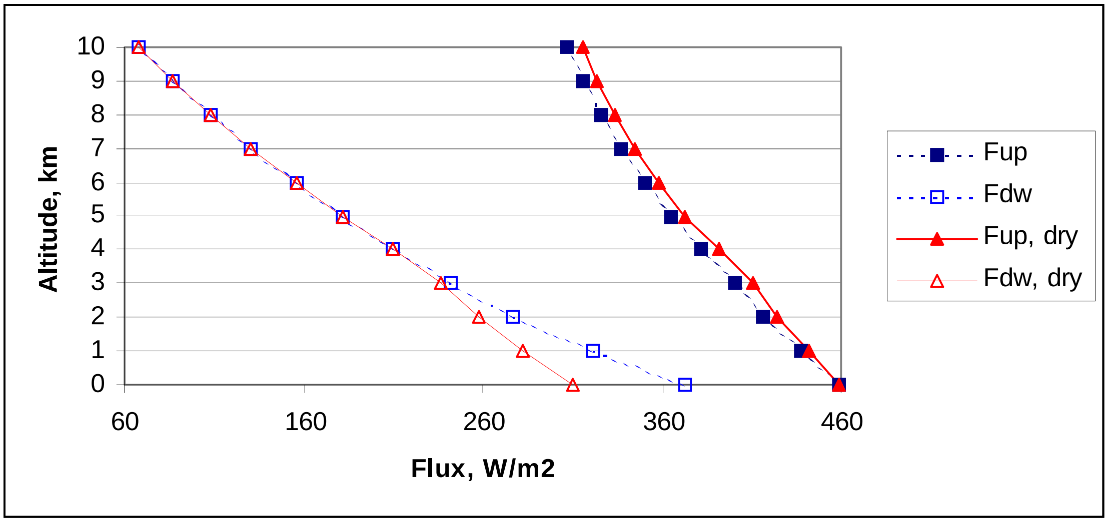
$$\left(\frac{dT}{dt}\right)_{IR} = -\frac{1}{c_p \rho}\frac{dF(z)}{dz}$$

<u>Effect of the varying amount of a gas on IR radiation under the same atmospheric condition</u>

Consider the standard tropical atmosphere and “dry” tropical atmosphere: same atmospheric characteristics, except the amount of H_2O

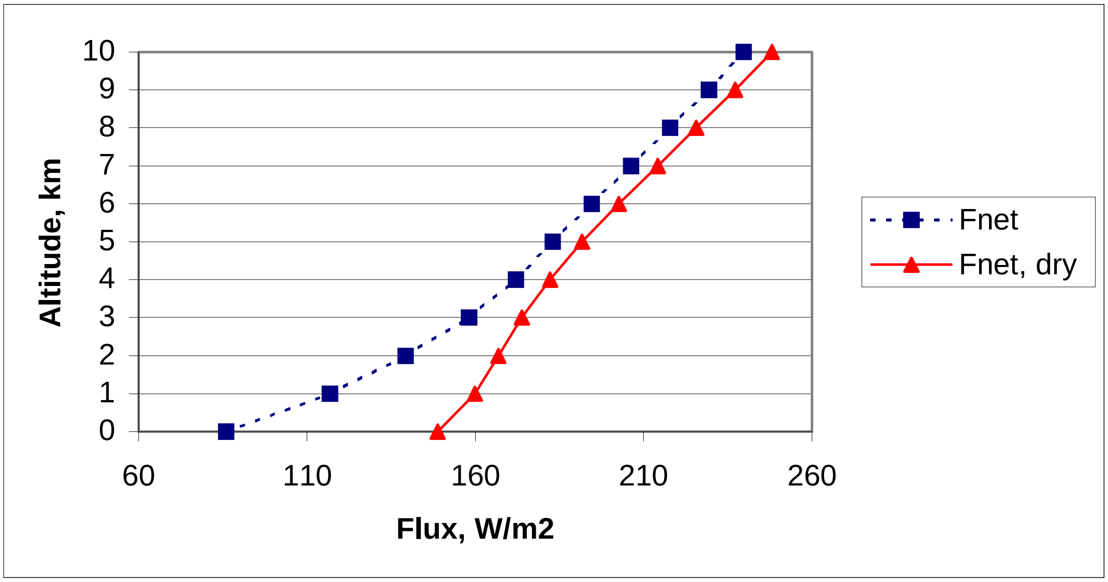


IR fluxes for tropical (dotted lines) and dry tropical atmospheres (solid lines)



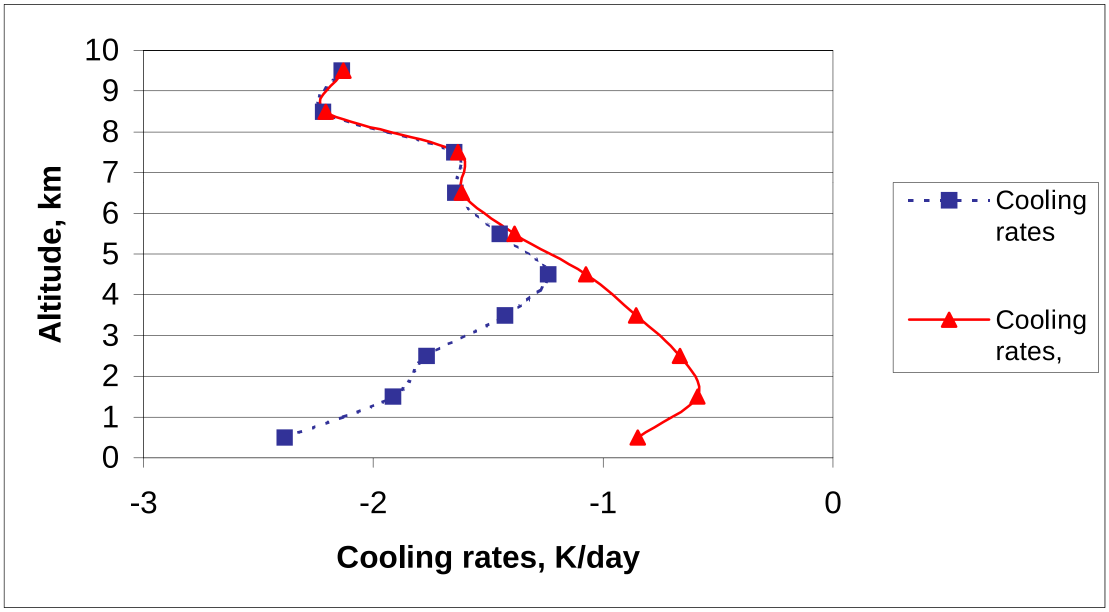
- H_2O increases in a layer => $F^{\downarrow}$ increases because more IR radiation emitted in a layer => $F^{\downarrow}(surface)$ increases
- H_2O increases in a layer => $F^{\uparrow}$ decreases because more IR radiation absorbed but reemitted at the lower temperature => $F^{\downarrow}(TOA)$ decreases
- Increase of an IR absorbing gas contributes to the greenhouse effect

IR net fluxes for tropical (dotted lines) and dry tropical atmospheres (solid lines)



- The larger changes of net flux from one level to another (i.e., the larger slope of F(Z) vs Z), the larger heating/cooling rates

IR cooling rates for tropical (dotted lines) and dry tropical atmospheres (solid lines)



NOTE: The largest IR cooling rates for the standard tropical atmosphere are in the surface layer.

3. Concept of broadband flux emissivity

- The **broadband flux emissivity** approach allows calculation of infrared fluxes and heating/cooling rates utilizing the temperature in terms of the Stefan-Boltzmann law instead of the Planck function.

Based on Eq.[11.5 a, b], the total upward and downward fluxes in the path length u coordinates may be expressed as

$$F^{\uparrow}(u) = \int_0^{\infty} \pi B_\nu (T_s) T_\nu^f (u)\, d\nu$$

$$+ \int_0^{\infty} \int_0^{u} \pi B_\nu (T(u')) \frac{dT_\nu^f (u - u')}{du'} du' d\nu \qquad [11.9a]$$

and

$$F^{\downarrow}(u) = \int_0^{\infty} \int_{u^*}^{u} \pi B_\nu (T(u')) \frac{dT_\nu^f (u' - u)}{du'} du' d\nu \qquad [11.9b]$$

From the Stefan-Boltzmann law (see Lecture 3), we have

$$\int_0^{\infty} \pi B_\nu (T)\, d\nu = \sigma_B T^4$$

Let's define **the isothermal broadband emissivity** as

$$\varepsilon^f (u, T) = \frac{\int_0^{\infty} \pi B_\nu (T)(1 - T_\nu^f (u))\, d\nu}{\sigma_B T^4} \qquad [11.10]$$

Using the **isothermal broadband emissivity,** Eq.[11.9a, b] may be approximated as

$$\boxed{F^{\uparrow}(u) \cong \sigma_B T_s^4 (1 - \varepsilon^f (u, T_s)) - \int_0^{u} \sigma_B T^4 (u') \frac{d\varepsilon^f (u - u', T(u'))}{du} du'} \qquad [11.11a]$$

and

$$\boxed{F^{\downarrow}(u) \cong \int_u^{u^*} \sigma_B T^4 (u') \frac{d\varepsilon^f (u' - u, T(u'))}{du'} du' d\nu} \qquad [11.11b]$$

NOTE: If the **isothermal broadband emissivity** is known, the broadband fluxes and heating/cooling rates can be easily calculated from Eq.[11.11a, b].